A Technique for Estimating Axial Flow Compressor Potential Peak Efficiency and Related Performance

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A technique for the rapid numerical estimation of the potential peak efficiency and related performance of an axial flow compressor is presented, along with an empirical method for determining the stage entropy gradient associated with changes in blade row-wise aerodynamic and geometric parameters. This technique can rapidly provide information to allow a solution of the three-dimensional compressor flow equations to be performed. Classical off-design methods are discussed only to show the need for and applicability of the proposed technique in existing compressor calculation methods. Empirical evidence has been used to illustrate qualitatively and to define quantitatively the influence of selected primary geometric and aerodynamic parameters. Basic limitations, including recommendations for improving and expanding the technique, are discussed. A Fortran computer program was prepared to carry out the solution of the empirical relations, and computational results are compared with data for a 5-stage transonic compressor. Extensive calculations have shown the quality of these comparative results to be typical for wide ranges of the controlling parameters.

Nomenclature

c = blade chord length
D, E, F, G = functions of only one geometric parameter or operating condition each
h = radial blade length
L = hydraulic cascade factor
M = relative Mach number
q = velocity head
Q = typical performance parameter
R = relative blade Reynolds number, based on chord
T = Mach number factor
V = Reynolds number factor
ϵ = radial clearance
Δ = static pressure rise
θ = blade camber angle
n = stage polytropic efficiency with clearances
n_0 = stage polytropic efficiency without clearances
σ = chord to spacing ratio, solidity
ξ = cascade stagger angle, measured from axial direction

1. Introduction

FROM the several types of compressors available for service in the gas turbine powerplant, the axial flow machine has grown in use most rapidly in recent years. For the present purposes, a "compressor" is defined and restricted to be that portion of an axial-flow turbomachine which transforms shaft work into energy of the gas stream. Physically, it will be considered to consist of an alternating sequence of rotating and stationary airfoil cascades. Energy addition is accomplished in the rotor cascades, and the stream is redirected by the following stator before entry into the next rotor. This flow pattern is obviously three-dimensional, with all the complications of internal viscous flow and nonsteady effects.

The object of this paper is to show that, in spite of the formidable physical complexities, a reliable technique for the estimation of potential multistage compressor efficiency can be devised to permit conventional design methods to be applied with a reasonable probability of success. This technique leans heavily on 1) the establishment of a flow model from theoretically suggestive analogies and 2) empirical evaluation of necessary coefficients and exponents. Accordingly, an analytical technique is outlined, based upon simple analogies and empirical coefficients, which will serve to enable a first-order estimation of compressor efficiency potential and related performance, at design and off-design speed operation, and related gradients with primary geometric and aerodynamic parameters. Desirable additional consequences of this technique are that it is of a type that can aid in the proper selection of a nominal aerodynamic design point, as well as aid in collating existing data, while serving to facilitate improvements in the technique itself.

2. Approach and Procedure

A basic postulate in similitude theory is: "If the same set of conditions and governing equations determine the behavior of two systems and if these two systems have identical values of all parameters in these governing relations, then similar behavior is exhibited by the two systems, provided only that a solution uniqueness condition exists." It is further known that a simpler form of similarity can frequently be found by seeking new coordinates that reduce the number of independent variables in the governing equations. Usually such coordinates have been called similarity variables.

There is currently an abundance of what may be called "exact particular solutions" to the "complete" system of turbomachinery equations and conditions in the form of existing, operating, or tested compressors. The term "exact solution," in reference to the Navier-Stokes three-dimensional, viscous, unsteady equations, is used in referring to a physical compressor that has been tested, or is under consideration; and synonymously implied by the term "solution" is knowledge of the vector diagrams throughout, along with the intrinsic flow properties, such as temperature and pressure.

The "potential peak efficiency and related performance" is defined, in this technique, as the best that has been experimentally obtained for a similarly defined compressor stage. This does not mean that it will or must be obtained, but only that it has been similarly obtained; i.e., with a reasonable confidence level, it can be expected to be available if proper design techniques are employed. It is noted, in passing, that a designer need not choose to design at the peak efficiency point; this possibility is discussed in Appendix A of Ref. 1.

It is important to differentiate clearly between the procedure herein considered and another type of procedure in

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current use. The present objective is not to facilitate the prediction of off-design performance, although benefits along this line do accrue; but, rather, the object is more essentially a technique that will enable the prediction of the design performance itself. Once this design performance is estimated, i.e., a physically possible solution having been found, then other existing procedures (such as Ref. 2) or an offshoot of this procedure may be used to obtain any desired estimates of off-design performance.

Accordingly, to obtain a solution for an untested compressor that differs from some existing compressor, it is then necessary to normalize the governing equations, forming appropriate analogous mathematical models; apply the basic similitude postulate; supply missing estimates from data; formulate perturbational effects of the significant parameters; apply these perturbational effects to successive stages through the existing solution; and thus, within the perturbational accuracy, obtain a solution that represents the new compressor. An inherent implication, in using this approach, is that some solutions are physically possible, whereas others may not be; this is, in reality, an assumption that certainly seems to be borne out by test data. It is emphasized that the perturbational effects of Mach number and Reynolds number on performance are not for a fixed compressor geometry but rather are for a varying geometry at the condition defined as peak stage efficiency.

The present method depends upon the following reasonable, basic assumption: that the compressor steady-state aerodynamic performance is a function of its steady-state operating conditions and its geometry. Thus, a typical aerodynamic performance parameter, in principle, can be expressed explicitly as

$$Q = Q(D, E, F, G) \tag{1}$$

More explicitly, Eq. (1) is presumed to be

$$Q = D^d E^e F^f G^g \tag{2}$$

where, in general, each of the exponents are individually functions of the operating conditions and physical geometry, e.g.,

$$d = d(Q, D, E, F, G) \tag{3}$$

Ideally, a situation wherein the functional forms of the mathematical models used in Eq. (2) are correctly representative, and the functions are separable, leads to the simple criterion of constant exponents. If sufficient data were available, curve fitting of the exponents could deal with experimental error. Strictly speaking, a finite amount of data can yield only a hypothesis about the forms.

In order to arrive at the functional forms needed in Eq. (2), compressor passage losses, at the peak efficiency point, are analogized qualitatively with those of simple diffusers to yield a correlation with geometry and Reynolds number. These diffuser losses are analogized, in turn, with those of an equivalent flat plate, bringing flat-plate theory to bear upon the analysis of compressors; a flat-plate correlation of Reynolds number and Mach number is then used to evaluate the two Reynolds number correlations, to imply applicability of the flat-plate Mach number correlation to compressor losses. Reference 1 contains details of the application of this procedure and the stated analogies to Eq. (2), along with the introduction of empirical data to evaluate exponents and coefficients. Some of the more explicit results for subsonic flow are summarized below:

$$n = n_0 e^{[-2.87\epsilon\cos\xi/(h\sigma)]} \tag{4}$$

$$[(1 - n_0)/0.14n_0]^2 = L^{0.75}/[V^{1.25}T^{0.226}] \tag{5}$$

$$\Delta/q = 2.63(n/n_0)^2 L^{0.5}/[\sigma^{0.125}V^{1.333}T^{0.241}] \tag{6}$$

where

$$L = [c/h + \sigma(1 - \epsilon/h)]/[\cos\xi \cos^2\theta] \tag{7}$$

$$V = \log_{10}[R/(\sigma^{0.25}\cos^2\theta)] \tag{8}$$

$$T = \{1 + [(\gamma - 1)/2]M^2\} \tag{9}$$

3. Applications of Technique

The indicated object of the present work is to present a technique capable of estimating the peak attainable stage efficiency. With this in mind, it is pointed out that the correlation, represented by Eqs. (4–9), must consider the flow throughout the entire passage, i.e., the correlation parameters should be considered and treated as average stage quantities. For instance, the stage static pressure rise coefficient Δ/q, is considered to be the integrated or mass weighted enthalpy rise normalized by the sum of the integrated blade row kinetic energies for a stage rotor and stator. This is over and above any rise caused by simple radius change across a rotor. Similarly, the stage efficiency is an average value.

In a similar fashion, the correlation factor should be summed over both the rotor and stator and averaged for a stage value. These summation signs have been omitted for two reasons: first, their inclusion does more to confuse than to clarify the expressions, and, second, including them would infer that the empirical coefficients were obtained by actually integrating the data of Refs. 2–4. A true integration would require far more detail than normally is reported or even measured. In practice, this type of integration can be carried out only during a design phase, when this detailed information is readily available.

Although it is true that the information necessary for a kinetic energy weighted integration can be simulated from relatively scanty data, it is questionable whether this additional effort is warranted in view of the relatively small qualitative differences that arise between the integral form and a simple pitchline form, especially in view of the assumptions involved in the said simulation. Nevertheless, it is believed that the correlation should be considered to be an integral average; the empirical coefficients in these equations were evaluated on a pitchline basis for expediency and consistency. Satisfactory pitchline results can be interpreted to mean, first, that the correlation technique is a proper avenue of theoretical consideration, and, second, that the correlation has useful validity even in its present form.

One procedure, then, is to use the correlation to relate the geometry to the aerodynamics and vice versa, in a manner similar to stage characteristics calculations. A speed, flow, stator schedule, design geometry and performance data at the peak efficiency condition are enough information to compute the actual stage efficiency and Δ/q, the correlation parameters, and the corresponding optimum performance. The ratio of the measured efficiency and Δ/q to that predicted by the correlation is a measure of how effectively the stage design compares with the optimum attainable.

Conversely, if no data are available, the correlation can be employed iteratively to arrive at the performance corresponding to the optimum at the peak efficiency point. The sequence can be summarized as follows: given a speed, flow, stator schedule, and geometry, the rotor inlet conditions can be directly computed. The correlation represented by Eqs. (4–9) is then used to estimate the conditions at the stage exit, allowing recalculation of the rotor exit conditions. This procedure is repeated within a stage until convergence on efficiency is accomplished. This, then, provides the inlet conditions to the following stage. The procedure is repeated, stage by stage, through the compressor, until the performance of the entire compressor is obtained.

This, of course, would correspond to a condition in which all stages were at their individual peak efficiencies, a situation that seldom occurs. At this point, consideration of the individual stage off-match would enter and the resulting individual stage performance would depend upon the degree of stage off-match. The estimation of individual stage off-match is discussed further in Ref. 1.

As stated in Ref. 1, if attention is directed at the map peak efficiency point, the individual stage off-match may be neglected for a first approximation since the over-all average stage off-match must be close to zero. The main consequence of omitting an off-match correction is to reasonably simulate the over-all performance although the individual stage performance might be in significant error. Far better internal results can be obtained by including the effects of off-match. The need for this off-match correction generally would increase with the number of stages.

4. Limitations and Numerical Example

A review of the technique, as represented by Eqs. (4–9), reveals and points out that these equations were obtained from essentially incompressible data, for which the Mach number effects could be removed from consideration. The Mach number effects were then reintroduced by hypothesizing that the concept of the equivalence of Mach number and Reynolds number effects on the duct frictional factor is applicable to diffuser passages. Although this is a reasonable first estimate, it is quite probable that a more detailed analysis would modify this result somewhat, although the effect of Mach number should remain qualitatively related to the Reynolds number effect. The possible additional supersonic Mach number effects are discussed in Ref. 1, and are omitted from consideration herein.

The indicated Reynolds number effect should be considered to be somewhat in the nature of a moderate to high Reynolds number effect; since the evaluation was restricted to the high Reynolds number, moderately high-flow-coefficient regime. Further, no second-order consideration has been given to multistaging effects, which would be in the nature of a turbulence level effect. Still further, it is inconceivable that there would be no effects of Reynolds number, vector diagram, or blade shape upon the alleged form of the clearance effect. Although much experimental data would suggest that these second-order clearance effects are small, they should be considered.

First-order effects of weight flow, wheel speed, and flow path are considered in the "stage characteristics" approach, but second-order effects of the flow coefficient regime, radius ratio, and vector diagrams are not completely considered. Their individual effects should be discernible, and could be considered to affect primarily the selection of a representative pitchline.

It should be noted that this correlation is based upon the concept of an optimum efficiency point; a particular stage may not actually simultaneously deliver the correlated pressure rise and efficiency even after correcting for stage off-match. There are two main reasons for this; conceptually, the technique would imply that the stage was designed either with a deliberate or an unintensional performance compromise and, practically, shortcomings in the technique itself could appear in this form. Conceptually, the technique requires that the empirical coefficients be such that the actual performance is equal to or less than the correlated estimate, at zero off-match.

After having obtained the empirical coefficients shown in Eqs. (4–9), a Fortran computer program (see Ref. 1 for general flow-chart) was written to handle the cumbersome calculations. The evaluation of a high-speed compressor with this technique necessitated including real gas effects. The technique was evaluated by examining the NACA five-stage transonic compressor[5–7] at the 90% speed peak operating efficiency point, which is close to the absolute peak efficiency that actually occurred at about 86% speed. The 90% speed first rotor tip relative Mach number is about 1.05. This data point is then close to the peak efficiency point whereas the individual stage off-match should be relatively small. It also is at a high Mach number in contrast to the essentially incompressible data of Refs. 2–4, from which the empirical constants of Eqs. (4–9) were evaluated. At the same time, the Mach number is sufficiently low to be essentially subsonic. This removes almost all supersonic shock losses from consideration, and allows an evaluation of the basic correlation without shocks. This compressor is operating at a very high Reynolds number (almost 2×10^6) in contrast to the low Reynolds number data used to evaluate the empirical coefficients.

The ratios of the actual stage efficiency to the values estimated by this technique were 0.972 ± 0.003. It is pointed out that no effort was made to modify the coefficients to improve the accuracy of this ratio; however, it is clear, first, that this can be done, and, second, that more evaluation of the results should be made before attempting to modify the coefficients or the form of the correlation. The average of the ratios of the actual stage Δ/q to the values estimated by this procedure was 0.99. The individual stage off-match was approximated in Ref. 1; the average was "-0.02," consistent with the assumption that, near the peak efficiency, the average stage off-match is near a minimum.

The extremely good results obtained in this example are quite encouraging. However, the only purpose of this example is to illustrate an extreme case wherein no data would be available other than the original data of Refs. 2–4. If the problem were the estimation of the performance of a compressor similar to the NACA five-stage it would be preferable to employ these NACA five-stage ratios of efficiency and Δ/q as estimates. Thus, the proposed technique can best be used as a perturbational technique until such time as improvements can be made in the correlation parameters and empirical coefficients.

5. Conclusions

A perturbational technique based on a hydraulic analogy has been devised which is capable of estimating the optimum efficiency performance of an axial flow compressor from a description of its geometry and aerodynamic environment. The demonstrated agreement of Eqs. (4–9) with high-speed data is quite encouraging. Although there are some limitations to the technique in its present state, it is felt that the basic technique of reducing comparative performance to a common basis for comparison can be a valuable aid in the analysis of compressors. It can be used to check quickly other procedures, as well as to provide basic information needed in other methods, such as the type described in Ref. 8. It has the further advantage of considering all forms of compressor losses on a perturbational basis.

Since the proposed technique is of a type that can empirically represent actual test data, it can serve the dual purpose of aiding in the interpolation of data as well as aiding in the collation of data. Thus, the technique is of a type that can be used judiciously to improve and extend itself as well as to evaluate even the validity of the form of the parameters used. It is felt that improving this technique to better understand actual radial and axial variations can lead to a correlation with reliability quite adequate for a practical, closed-form, three-dimensional numerical compressor calculation procedure. It is conceivable that further experience with this technique may suggest useful restricted solutions to the actual flow equations.

An extensive collation of test data undoubtedly would lead to some modifications of this technique, the empirical coefficients, and the form of the parameters, to remove any systematic errors. Nevertheless, the present form of the technique offers a means of evaluating the need for any of these modifications. The encouraging results obtained with this hydraulic analogy suggest further effort toward the rigorous analysis of secondary, viscous, curved-pipe flow as a possible means of better understanding cascade flow.

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Hydrodynamics of Tire Hydroplaning

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Tire hydroplaning is treated theoretically solely from a hydrodynamical standpoint. Curved planing surfaces of arbitrary shape are incorporated in the theory to simulate the wetted portion of a hydroplaning tire. The fluid is assumed to be ideal and to be undergoing irrotational, two-dimensional motion. The lift force on the planing surface and the pressure distribution on the runway surface are compared with experimental results of NASA. For the initial lift-off condition of incipient hydroplaning the lift coefficient from the theory is 0.8. The corresponding experiment value is 0.7. The pressure distribution on the runway surface from theory compares favorably with experiment both in shape and in magnitude.

Nomenclature

a = location of point F in t plane
A = characteristic area in equation for lift force
A_0, A_n = Euler coefficients for Fourier series
b = location of point D in t plane
B = complex constant
C_L = lift coefficient
C_{Li} = lift coefficient corresponding to incipient hydroplaning
C_p = pressure coefficient $(p - p_0)/(\rho U^2/2)$
d = resulting water depth downstream from tire
D = initial water depth on runway
$f(\phi_1), g(\phi_1)$ = arbitrary functions that describe planing surface
F_L = lift force on tire or planing surface
h = clearance between planing surface and runway surface
i = $(-1)^{1/2}$
k = elliptic modulus of Jacobian elliptic function
K = complete elliptic integral of first kind
K' = associated complete elliptic integral of first kind
l = total length of planing surface
n = $1, 2, 3, \ldots$
p = fluid (water) pressure at a point
p_i = tire-inflation pressure
p_0 = reference fluid pressure
q = Jacobi's nome $(e^{-\pi K'/K})$
t = intermediate complex plane
u, v = horizontal and vertical velocity components, respectively
U = translational speed of tire or speed of jet
V = total fluid velocity at a point
w = complex-potential $(\phi + i\psi)$
w_1 = auxiliary complex-potential $(\phi_1 + i\psi_1)$
x, y = horizontal and vertical coordinates, respectively
z = physical plane $(x + iy)$
δ = angle of fluid velocity vector
ζ = complex velocity $(1/U)(dw/dz)$
H = eta function of Jacobi
θ = angle flat plate makes with approaching jet
Θ = theta function of Jacobi
λ = parameter used to describe planing surface
ρ = mass density of fluid
σ = parameter used to describe planing surface
ϕ = velocity potential
ϕ_1 = auxiliary function
ψ, ψ_1 = stream and auxiliary functions, respectively
Ω = logarithm of complex velocity, $\ln[(1/U)(dw/dz)]$

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Introduction

THE planing of an aircraft tire on a flooded pavement is well known to pilots. The phenomenon is called tire hydroplaning, as it results from the water pressures developed between the tire and pavement surface. Hydroplaning occurs for an aircraft tire at the instant the total hydrodynamic force is equal to the load on the individual wheel. The tire then actually loses contact with the runway surface and essentially skis on the water. The preponderance of the experimental work concerning this phenomenon has been